Name: Solutions ____________ / 25

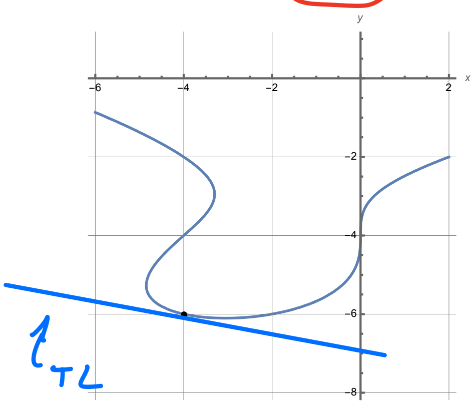
There are 25 points possible on this quiz. *You should be able to complete it without using your notes or textbook – this is practice for your exams!* If you needed to look something up, you should talk to me about questions you might have. **Show all work for full credit** and use some words or sentences to help communicate your answers. **Do not use a calculator.** No aids (book, calculator, etc.) are permitted.

1. **[8 points]** Find the equation of the tangent line to the implicitly defined function

$$x^2 - (y+4)^3 = xy$$

(-4, -6) whoops

at the point $P = (-4, 6)$ and sketch the tangent line on the graph. Clearly show your work.



$$2x - 3(y+4)^2 \frac{dy}{dx} = x\frac{dy}{dx} + y \Rightarrow$$

$$\frac{dy}{dx}\left(-3(y+4)^2 - x\right) = y - 2x \Rightarrow$$

$$\frac{dy}{dx} = \frac{y-2x}{-3(y+4)^2 - x}.$$ At $(x,y) = (-4,-6)$,

$$\frac{dy}{dx} = \frac{(-6)-2(-4)}{-3(-6+4)^2-(-4)} = \frac{-6+8}{-3(-2)^2+4} = \frac{2}{-12+4} = \frac{-2}{8} = -1/4$$

Equation of tangent line: $y = -\frac{1}{4}(x+4) + 6$

2. **[5 points]** Use **logarithmic differentiation** to find the derivative of

$$f(x) = (x^2 - 4x)^{3x}.$$

Let $y = f(x)$. Then $\ln(y) = \ln\left((x^2-4x)^{3x}\right) = 3x\ln(x^2-4x)$

So $\frac{y'}{y} = 3x\left(\frac{1}{x^2-4x}\right)(2x-4) + 3\ln(x^2-4x)$

$$\Rightarrow y' = \left(\frac{3x(2x-4)}{x^2-4x} + 3\ln(x^2-4x)\right)\underbrace{(x^2-4x)^{3x}}_{y}$$

$$= \left(\frac{3(2x-4)}{x-4} + 3\ln(x^2-4x)\right)(x^2-4x)^{3x}$$

3. [12 points] Find the derivative for each function below. Use whatever technique you like. $\boxed{\text{Do not simplify}}$. You do need to use parentheses correctly.

a. $h(x) = \frac{1}{x} + \ln(x)$ $= x^{-1} + \ln(x)$

$h'(x) = -x^{-2} + \frac{1}{x} = \frac{-1}{x^2} + \frac{1}{x} = \frac{-x+1}{x^2}$

b. $f(x) = \arcsin(7x)\left(\frac{1}{x^3}\right)$ $= \arcsin(7x) \cdot x^{-3}$

$f'(x) = \arcsin(7x)(-3x^{-4}) + x^{-3}\left(\frac{1}{\sqrt{1-(7x)^2}}\right)(7)$

Note $\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$

c. $y = (2^x + \arctan(x))^5$

$y' = 5(2^x + \arctan(x))^4\left(2^x \ln(2) + \frac{1}{1+x^2}\right)$

d. $g(x) = \dfrac{x^5 - 2}{e^{7x+6}}$

$$g'(x) = \frac{e^{7x+6}(5x^4) - (x^5-2)(e^{7x+6}(7))}{(e^{7x+6})^2}$$